THEORY OF ELEMENTARY PARTICLES

Tokuichi Tajima.

The following paper is based on the interaction and intends to establish a theory into which we introduce an operator represented by

$$H = F + iI \qquad (i = \sqrt{-1}),$$

which is formed by two non-commutable hermite operators F and I.

In this paper we deal with the new space of this theory and with a general discussion in a philosophical standpoint of the problems arising in this new theory.

§ 1. Fundamental Principle.

Absolutes are the characteristic of a classical physics. There are aspects of all sorts in them.

As the modern physics, relativity and quantum theory, advances, these **Absolutes** are conditioned one after another.

From this point of view we introduce a fundamental principle,

"There are no Absolutes in nature."

This means also that :

> **Absolutes** can stand only under certain conditions that are produced by the influences of all fields for all elements of **nature**. (Table 1 .)[#1.]

Here we mean by the word of **nature** the description of it through the mathematical laws, i.e. the differential equations in the system which is determined by the measurement of physical quantities——the dynamical variables and the dynamical quantities——and by that of the state having these physical quantities in the relativistic 4-dimentional space-time coordinate.

Elements of Nature	Condition.
Mathematical Law. — Quantity.	Unit and Coordinate. (Mathematical Law is Variations of these Combinations.)
Mathematical Law. — Quality.	L. de Broglie's Material Wave.
Measurement.	Heisenberg's Uncertaintity Principle.
Physical Quantity. — Dynamical Variable.	Bohr's Complementarity.
Physical Quantity. — Dynamical Quantity.	The *unknown* to us. (Our theory.)
State.	Dirac's Superposition Principle.
Space.	Einstein's Relativity Theory.

Table 1.

There are in the old theory three **Absolutes** other than **Absolute** in Table 1, which do not satisfy our principle.

The first one is the field operator which refers to any specified particle, the second one is the localization of the field and of the interaction, and the third one is the causality.

In respect to the first, the field operator, Heisenberg has made researches in detail. [#2] Refering the field operator to any specified particle like proton,meson,etc.,is contradictory to our principle. But Heisenberg's procedure is not the unique expression satisfying our principle and our principle does not seem to lead to his non-linear equation.

The second and the third **Absolute** have been studed by Yukawa. [#3] We may use his results when we construct the concrete form of H , but we must rearrange them.

No Absolute does not necessarily mean an intermediate state. In observation all measured values must take a unique value and it is impossible for them to have a dual value or an intermediate value. This means to attach a condition to the observation or to take the field into consideration. [#4]

Developments in contemporary physics are perhaps moving slowly toward the tacit removal of this conditioned **Absolute** but the developments are without conscious effort in this direction and their fruits are now unknown to us. But to construct a theory fitting for this principle is reasonable and might have much benefit. [#5] Moreover I believe this would be useful for removing the difficulties in recent field theory.

This principle does not contradict the principles of relativity or of quantum theory. It is also consistent with the general standpoint of physics. Hence we have to construct our theory for fitting this principle for all elements in the description of nature and in all steps of the treatment. [#6]

At first we shall take up the dynamical quantity, namely Hamiltonian. [#7]

§ 2. Non-Hermite Operator.

In the case of the dynamical quantity we cannot express the condition of **Absolutes** by the non-commutability of them, because the dynamical quantity has not the direct non-commutative quantities and the commutation relation is not indicative for the non-commutability of the dynamical quantities. The relation of the non-commutability of two dynamical quantities is only the other expression of the non-commutability of the dynamical variables constructing them.

The old Hamiltonian is generally classified into two kinds, the free Hamiltonian H_o and the interaction Hamiltonian H_{in}. We completely ignore in the free state the self-field where the particles exist and all the other interactions, for the free state is produced as a result of neglecting all the influences. This is really the **Absolute** of the dynamical quantity that is clearly contradictory to our principle. [#8]

The foundation of the old theory was built in this absolute situation and the interaction theory was only extended and analogized through this free theory. The transition from the later to the former was only possible when the transition from the free to the interaction and the contrary were continuously possible, but what if the free had been essentially special? We cannot manage only the question, whether we have no rigorous solution in mathematics in the case where the interaction exists.

I think it proper to consider the free as the special case of the interaction, the interaction being the basis of the theory. We should have made a start from the interaction. Indeed the interaction is more fundamental and the free must be the special case.

Hence the new free in our theory is a conditioned form of the basic system and is essentially different from the interaction. But our free theory is identical with the old one.

In the case of the old operator, two terms F and I are combined generally as follows :

$$H_{old} = F + I \qquad (1).$$

Hereafter we can not use the hermite operator as in (1) but we must extend it to a normal or, shall we say, a **non-hermite operator**. This normal or non-hermite operator H is

$$H = F + iI \quad (i = \sqrt{-1}) . \qquad (2).$$

It is namely the combination of two hermite operators F and I.

When F and I are commutable ($F \vee I$), H in (2) it becomes the normal operator and when they are non-commutable ($F \not\vee I$) it becomes the non-hermite operator. In the concrete case, let F be a free Hamiltonian H_o of the hermite operator and let I be the same hermite interaction Hamiltonian H_{in}, then

$$H = H_o + iH_{in} \qquad (3).$$

satisfies our principle and is what we pursue. [#9] This construction does not contradict the principles of the relativity theory and of the quantum theory.

When H is changed into the normal operator its properties are almost the same as the hermite itself, but when H turns into the non-hermite operator this simplicity is broken and we do not know at present its properties even in mathematics.

When we neglect I, namely when (2) turns into F, the newmade type of H i.e. F, is essentially different from that of H which is composed of F and I. In other words, F is the hermite operator, where H is the non-hermite operator.

Even in the case of an old Hamiltonian shown by

$$H_{old} = H_o + H_{in} \qquad (4).$$

the interaction is naturally taken into consideration and this old theory concurrently treats the interaction with the free but it does not at all mean that the both terms, H_o and H_{in}, must be complementary. H_o and H_{in} are of the same quality, i.e. the hermite operators, and each is the same as H_{old} i.e. the hermite operator. Hence, the **Absolute** system where H_o stands apart from the other is entitled to an existence in nature and coincides with the usual reality, namely the actuality having the interaction whether the interaction may exist or not.

On the contrary, in the case of (3), H is essential and H_o is different from H in quality. Hence we can not neglect H_o or H_{in} at all, that is to say H_o and H_{in} cannot be isolated without changing the content of the space.

Our principle indicates that it is absolutely necessary that H expressed in (2), namely always accompanied by the interaction, must be the operator.

Such a concurrent treatment of the interaction with the free is physically one of the most important keys in this theory.

We need not consider the case where the interaction Hamiltonian is constructed to hold i at the outset, — that is the case when H_{in} is not the hermite operator — but it might also be possible to insert i into place other than in front of the interaction Hamiltonian, H_{in}. We might insert i into the free term and let the interaction term be a real part or we might insert i into any part of a free Hamiltonian and at the same time into any part of the interaction Hamiltonian. Where no clear difference exists between the free and the interaction term, we are, of course, unable to make a choice. These possibilities will be eliminated, however, by our demand that i be inserted into that part which allows complete conformity with the case of the old free theory when the imaginary part is extracted.

The interaction term and the free term in the Hamiltonian generally do not mutually commute. Of course, if we adopt a special interaction term it can be commutable with the free term, but this case will have physically no meaning. It is thus sufficient to take up only the non-hermite operator as the operator.

The non-hermite operator (3), which has the same properties as the algebra of the normal operator except for the commutability of the product between H_o and H_{in}, is the generalized operator including the hermite operator and the normal operator (or the unitary operator).

To sum up, we must use the dynamical quantity constructed from the dynamical variables as a non-hermite operator so that we may satisfy our principle.

Note. : From our field equation for this non-hermite operator (2),

$$\left.\begin{array}{l} F\Psi = f\Psi \\ I\Psi' = j\Psi' \end{array}\right\} \qquad (5).$$

$f+ij$ is not immediately equal to the measured real value λ.

λ is obtained by

$$\lambda = \mathbf{OP.}(f+ij) \qquad (6).$$

where **OP.** denotes a definite operator to make complex $f+ij$ real λ.

§ 3. Space.

In order to indicate the difference, we shall call the space in the case of the hermite operator the **Hermite Hilbert Space** (H.H.) that of the anti-hermite operator the **Anti-**

Hermite Hilbert Space (A. H.) and that in the case of the normal operator the **Normal Hilbert Space** (N. H.).[#10] Furthermore, we shall call the space produced by the non-hermite operator the **Non-Hermite Hilbert Space** (or the **2n-dimentional Orthogonal Doublet Hilbert Space**) (D. H.).

As far as the N. H. produced by the diagonalization of the unitary operator is concerned, there is no essential difference from usual Hilbert space. But when we have extended the operator to the non-hermite operator, we cannot solve the eigenvalue problem by the ordinary normed-orthogonal system.

At first in the case of

$$I=O \qquad (7).$$

the non-hermite operator (2) in D. H. changes into the hermite operator F,

$$H=F \qquad (8).$$

and a space produced by this change will be H.H., the operator of which is diagonalized by the unitary operator U,

$$U^{*} U=UU^{*}=1. \qquad (9).$$

In the following case

$$F=0 \qquad (10).$$

namely when H turns into iI,

$$H=iI \qquad (11).$$

this space becomes A. H. constructed by the operator, the property of which is

$$H^{*}=-H. \qquad (12).$$

Then from this argument, we conclude that both spaces formed by the only free part and the only interaction part are the same in their properties as the usual Hilbert space.

On the other hand, the space that must always have the interaction is the D. H. entirely different from the above mentioned space in structure.

The system does not make the n-dimentional normed-orthogonal one and therefore this space does not make the ordinary Hilbert space. As we had previously explained, D. H. is the space where the real H. H. produced from the F-part is combined with the imaginarly H. H. produced from the I-part. So this space is formed from the 2n-dimentional orthogonal doublet Hildert space. Real F-H. H. and imaginarly I-H. H. coexist and the both spaces are crossed obliqely each other. By the relation

$$FI-IF=il, \quad l=\text{real const.} \qquad (13).$$

both spaces are combined. When this relation (13) turns to

$$FI-IF=O \qquad (14).$$

this D. H. moves to the ordinarly N. H. The D. H. is the very space having these deformations from H. H.

The S -matrix is given by

$$S=\sum_{n=0}^{\infty}\frac{1}{n!}\left(-\frac{ie}{\hbar c}\right)\int_{-\infty}^{\infty}d^4x_1\cdots\int_{-\infty}^{\infty}d^4x_n\, P(F(x_1)+iI(x_1),\cdots F(x_n)+iI(x_n)) \qquad (15).$$

similar to Dyson's S-matrix using the chronological notation.

This S is furthermore a normal operator

$$S^*S=SS^*. \qquad (16).$$

This space is then N. H. containing H. H. (Fig. 1).

To make the separation of two particles ∞ is the same as to make the interaction infinitely small. Namely when we take up the S-matrix, its space is N. H. and in its limiting case (7) the space will be H. H.

Moreover, to make the case (7) is the same as to make the real-H. H. in D. H. and the case (10) is therefore to make the imaginarly-H. H. in D. H.

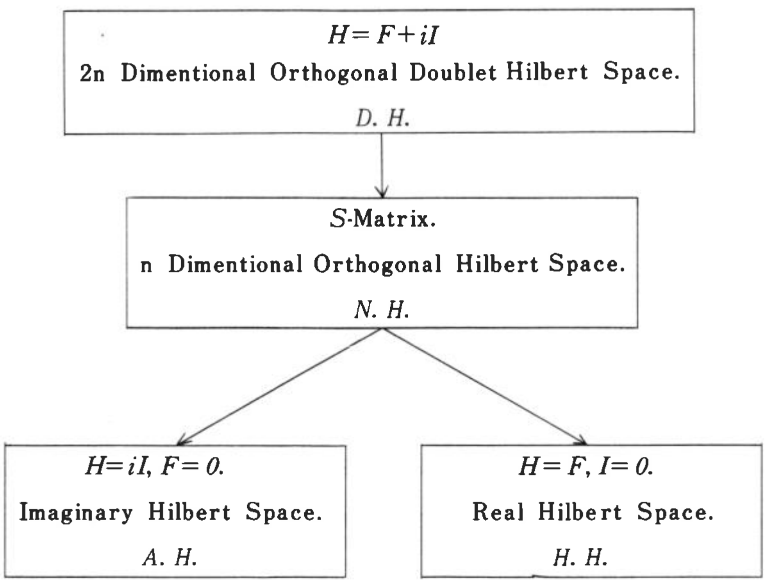

Fig. 1.

The correction added to the theory of H. H. on the occasion of the extension into D. H. returns to its origin in H. H. when we return to the hermite operator from the non-hermite operator H. In order to justify this facts it is sufficient to show that the theory of (8) eliminating the iI part coincides with the old theory of H. H.

Hence the D. H. is the space which includes H. H.

#1. Hereafter we use Absolute in the sense of a recognition of the existence without condition.

#2. W. Heisenberg : Revs. Mod. Phys., **29**. (1957), 269.; etc.

#3. H. Yukawa : Phys. Rev., **76**. (1947), 300.;

77. (1950), 219.; etc.

#4. See § 2 Note in this paper.

#5. The import of this principle is that all subjects of nature have to deal with a solid configuration and the description of it is only one of the plane section of its configuration.

#6. In the future we must proceed by unificative method rather than by a consideration of the separate elements of description and the correlation between them will be problematic. Only when all these studies are over we can express our principle by mathematical formula.

#7. In order to make the physical meaning clear we take up especially the Hamiltonian as the dynamical quantity. Our theory is fitting for all operators, Hamiltonian, Lagrangian and others.

#8. Neither has the new conception of physics been brought into the theory by the hermitalization nor the hermicity is necessary in either the principle of relativity or that of quantum theory.

#9. At the first step of this study we established the following conditions that H_o be equal to the old free Hamiltonian and that H_{in} be the same as the old interaction hamiltonian But we may take the newmade H_o and H_{in} as analogous to these old terms.

If H_{in} is not the hermite operator from the outset, H should be constructed not as in (3) but as in (1). For example, when H_{in} is given by

$$H_{in} = H^1_{in} + iH^2_{in},$$

where H^1_{in} and H^2_{in} are the hermite operators, H will be given by

$$H = H_o + H_{in}.$$

#10. The structure, the property and the difference of the space are already shown in many text book of mathematics.